MATH-904 Partial Differential Equations-II

Credit Hours: 3-0

Prerequisite: MATH 903 Partial Differential Equations-I

Course Objectives: The aim of the course is to motivate students to study different topics of the theory of partial differential equations. The course provides an overview on different topics of the theory of partial differential equations, Wave Equation—Properties of Solutions, The Notion of Energy of Solutions, Phase Space Analysis for the Heat Equation, Phase Space Analysis for Wave Models.

Core Contents: Basics for Partial Differential Equations, The Cauchy-Kovalevskaja Theorem, Holmgren's Uniqueness Theorem, Method of Characteristics, Burgers' Equation, Laplace Equation- Properties of Solutions. Heat Equation—Properties of Solutions.

Detailed Course Contents: Classification of Linear Partial Differential Equations of Kovalevskian Type, Classification of Linear Partial Differential Equations of Second Order, Classification of Linear Systems of Partial Differential Equations, Classification of Domains and Statement of Problems, Classification of Solutions. Classical Version, Abstract Version, Applications of the Abstract Cauchy- Kovalevskaja Theorem. Classical Version, Abstract Version, Quasilinear Partial Differential Equations of First Order, The Notion of Characteristics: Relation to Systems of Ordinary Differential Equations. Influence of the Initial Condition, Application of the Inverse Function Theorem. Classical Burgers' Equation, Other Models Related to Burgers' Equation. Poisson Integral Formula, Properties of Harmonic Functions, Other Properties of Elliptic Operators or Elliptic Equations. Boundary Value Problems of Potential Theory, Potential Theory and Representation Formula, Maximum-Minimum Principle. Qualitative Properties of Solutions of the Cauchy Problem for the Heat Equation, Mixed Problems for the Heat Equation d'Alembert's Representation in R, Wave Models with Sources or Sinks, Kirchhoff's Representations. Propagation of Singularities. Energies for Solutions to the Wave Equation, Examples of Energies for Other Models**.** Behavior of Local Energies. The Classical Heat Equation, The Classical Heat Equation with Mass, The Classical Wave Model, The Classical Damped Wave Model, Viscoelastic Damped Wave Model, Klein-Gordon Model

Course Outcomes: Students are expected to understand different topics of the theory of partial differential equations such as: The Cauchy-Kovalevskaja Theorem, Holmgren's Uniqueness Theorem, Method of Characteristics, Burgers' Equation, Laplace Equation- Properties of Solutions. Heat Equation—Properties of Solutions.

Text Book: Marcelo R. Ebert, Michael Reissig, Methods for Partial Differential Equations, Springer International Publishing AG 2018.

Reference Books:

1. Fritz John, Partial Differential Equations, Springer-Verlag, 1978.
2. Michael Eugene Taylor, Partial Differential Equations I: Basic Theory, Springer-Verlag, 1996.
3. R.C. McOwen, Partial Differential Equations: Methods and Applications, Pearson, 2004.

Weekly Breakdown		
Week	***Section***	***Topics***
1	3.1-3.5	Basics for Partial Differential Equations: Classification of Linear Partial Differential Equations of Kovalevskian Type, Classification of Linear Partial Differential Equations of Second Order, Classification of Linear Systems of Partial Differential Equations, Classification of Domains and Statement of Problems, Classification of Solutions.
2	4.1-4.3	The Cauchy-Kovalevskaja Theorem: Classical Version, Abstract Version, Applications of the Abstract Cauchy-Kovalevskaja Theorem.
3	5.1,5.2	Holmgren's Uniqueness Theorem: Classical Version, Abstract Version,
4	6.1,6.2	Method of Characteristics: Quasilinear Partial Differential Equations of First Order, The Notion of Characteristics: Relation to Systems of Ordinary Differential Equations.
5	6.3, 6.4	Influence of the Initial Condition, Application of the Inverse Function Theorem.
6	7.1,7.2	Burgers' Equation: Classical Burgers' Equation, Other Models Related to Burgers' Equation.
7	8.1-8.3	Laplace Equation-Properties of Solutions: Poisson Integral Formula, Properties of Harmonic Functions, Other Properties of Elliptic Operators or Elliptic Equations.
8	8.4, 9.1,9.2	Boundary Value Problems of Potential Theory, Heat Equation-Properties of Solutions: Potential Theory and Representation Formula, Maximum-Minimum Principle.
9	**Mid Semester Exam**	
10	9.3, 9.4	Qualitative Properties of Solutions of the Cauchy Problem for the Heat Equation, Mixed Problems for the Heat Equation
11	10.1, 10.4	Wave Equation—Properties of Solutions: d'Alembert's Representation in R,

		Wave Models with Sources or Sinks, Kirchhoff's Representations.
12	10.6, 11.1, 11.2	Propagation of Singularities. The Notion of Energy of Solutions: Energies for Solutions to the Wave Equation, Examples of Energies for other Models.
13	11.3, 12.1,12.2	Behavior of Local Energies. Phase Space Analysis for the Heat Equation: The Classical Heat Equation, The Classical Heat Equation with Mass.
14	14.1, 14.2	Phase Space Analysis for Wave Models: The Classical Wave Model, The Classical Damped Wave Model,
15	14.3	Viscoelastic Damped Wave Model
16	14.4	Klein-Gordon Model
17	-	Review
18	**End Semester Exam**	